Addendum: Updated phosphorus retention model for Danish lakes

A note on statistical modeling of total phosphorus retention under steady-state assumptions

Scientific note from DCE – Danish Centre for Environment and Energy

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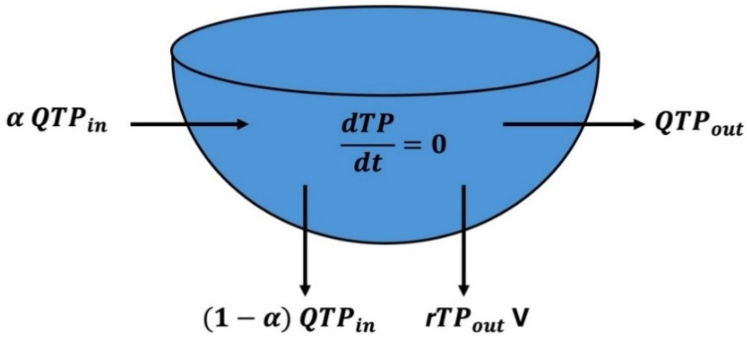

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Data sheet

Scientific note from DCE – Danish Centre for Environment and Energy

Category: Scientific briefing

Title: **Addendum:** Updated phosphorus retention model for Danish lakes
Subtitle: A note on statistical modeling of total phosphorus retention under steady-state assumptions

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1 Preface

This report addendum presents supplementary analyses and updates regarding the performance of the phosphorus retention model for Danish lakes. To concur with the current lake TP retention model used by SGAV, the output from Ladwig et al. (2026) was rearranged in this report to provide a TP retention coefficient normalized between 0 to 1. An exponential model using lake residence time was found to be the best predictor of the normalized TP retention coefficient.

2 Summary

The gross TP retention results from the original report (Ladwig et al. 2026) were reformulated to be expressed in normalized form in a range between 0 to 1 to be compatible with the sub-catchment outflow model by SGAV. Residence time was identified as the optimal covariate to calculate normalized gross TP retention in an exponential model:

All lakes:	$\lambda = 0.2280916\ t_w^{0.07298647}$
Shallow lakes:	$\lambda = 0.1981346\ t_w^{0.01245211}$
Deep lakes:	$\lambda = 0.2106041\ t_w^{0.2079472}$

with λ as normalized gross TP retention (dimensionless, between 0 to 1) and t_w as residence time (in years). Compared to Trolle et al. (2015), this updated model predicts lower gross TP retention values. The predicted lower retention by the updated model is probably due to (a) the fitting of the model to longer time series data, and (b) the incorporation of a TP loss term near the inlet. But, as the exponential model agrees well with the modeled gross TP retention (R^2 = 0.46 for all lakes), and the gross TP retention model achieved an RMSE of 0.038 mg TP L^{-1}, an NSE of 0.758, and an R^2 of 0.771 to observed TP, it can be assumed that observed TP retention in Danish lakes is lower than previously assumed.

3 Rationale

SGAV's sub-catchment outflow model applies a lake TP retention coefficient (normalized), $r_{lake,t}$, to quantify TP outflow depending on lake and time t as:

$$TP_{out,t} = (TP_{upstream,t} + c_t\, TP_{local,t})(1 - k_{lake,t}\, r_{lake,t})$$

Outflow TP concentration ($TP_{out,t}$) is the sum of inflow TP, $TP_{upstream,t}$, and local TP, $TP_{local,t}$, which is not retained by the lake system. Retention is here incorporated as $r_{lake,t}$, which is dimensionless in contrast to the gross TP retention in mass per time units as calculated in the original note, see Ladwig et al. (2026). This makes the straightforward integration of the derived gross TP retention into the sub-catchment model challenging.

Therefore, to concur with the current lake TP retention model used by SGAV, Eq. 7 from Ladwig et al. (2026) was rearranged from

$$ret_{gross} = rTP_{out}V + (1 - \alpha)QTP_{in}$$

to

$$\overline{ret_{gross}} = \frac{rTP_{out}V + (1 - \alpha)QTP_{in}}{QTP_{in}}$$

Here, $\overline{ret_{gross}}$ represents normalized gross TP retention as a fraction of total TP inflow load ranging from 0 to 1. Conceptually, this is identical to the one derived by Trolle et al. (2015), which was the ratio of in-lake TP to inflow TP.

All analyses are described in detail in Ladwig et al. (2026).

4 Results and Discussion

All performance metrics from Ladwig et al. (2026) for the statistical model to derive outflow TP concentrations are still valid as only the post-processed gross TP retention is re-arranged.

Normalized gross TP retention, $\overline{ret_{gross}}$, for most lakes is low with the average $\overline{ret_{gross}}$ of 0.21, maximum of 0.57, and minimum of 0.18 (Fig. 1).

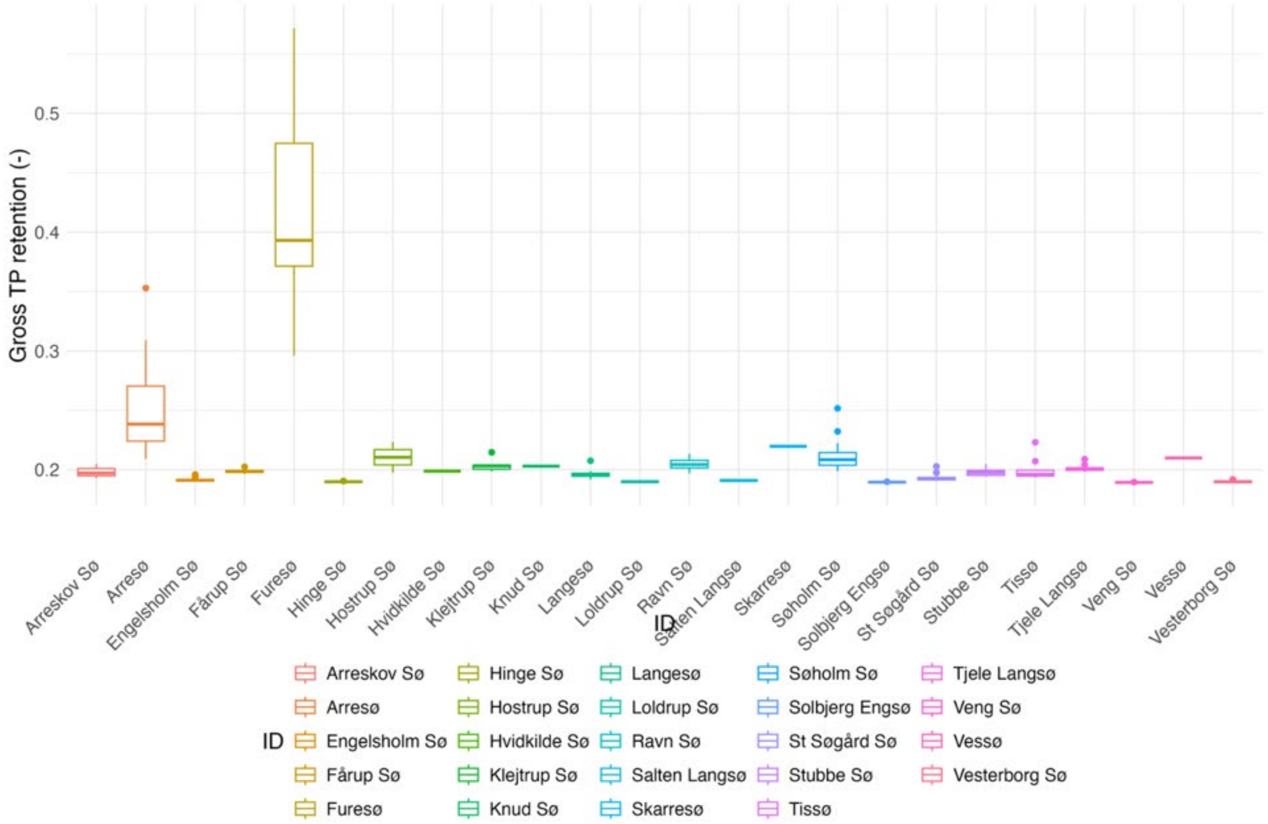


Figure 1. Boxplots of modeled normalized gross TP retention. In the boxplots, horizontal solid line indicates median of the distribution, the box represents the lower to upper quartile values of the data, the whiskers extend to the last data point beyond 1.5 * the interquartile range, circles represent outliers beyond this range.

Gross TP retention over time remained mostly constant in the model or decreased slightly over time (Fig. 2).

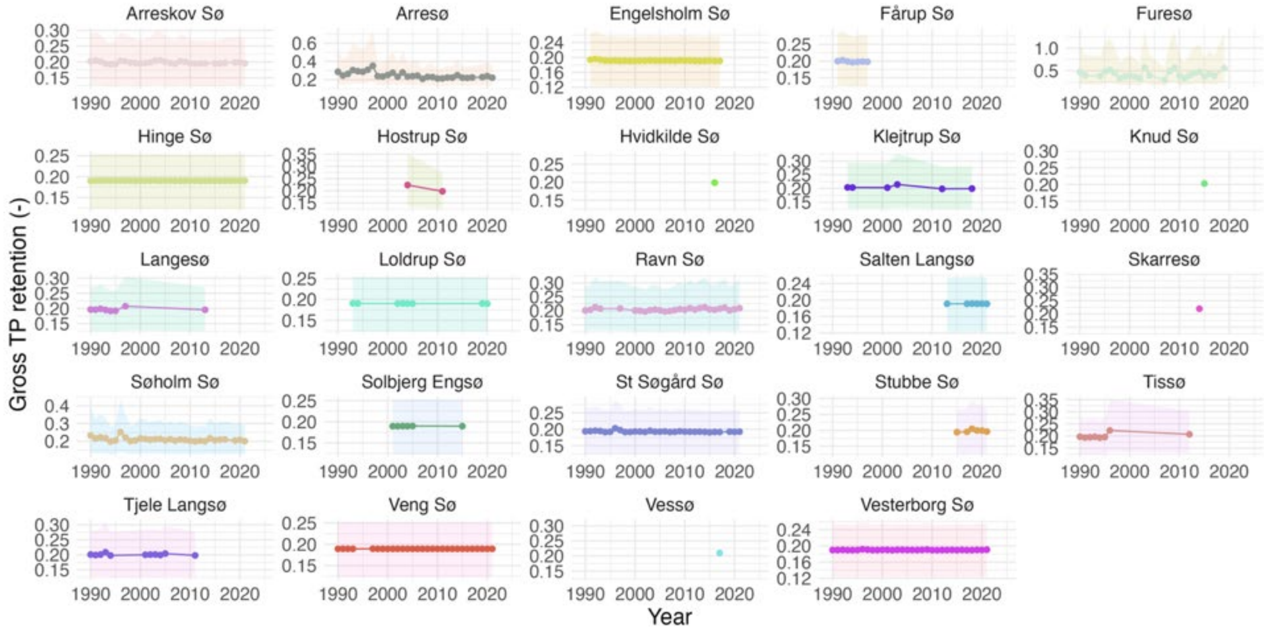


Figure 2. Gross TP retention over time. Shaded areas represent upper and lower constant bounds around the lake-specific average.

Boruta analysis highlighted that residence time, as in Trolle et al. (2015), is the most sensitive covariate to predict $\overline{ret_{gross}}$ (Fig. 3). As the focus is on the prediction and not inference, the use of residence time seems sufficient to build an exponential TP retention model, although the normalized gross TP retention itself is a function of residence time.

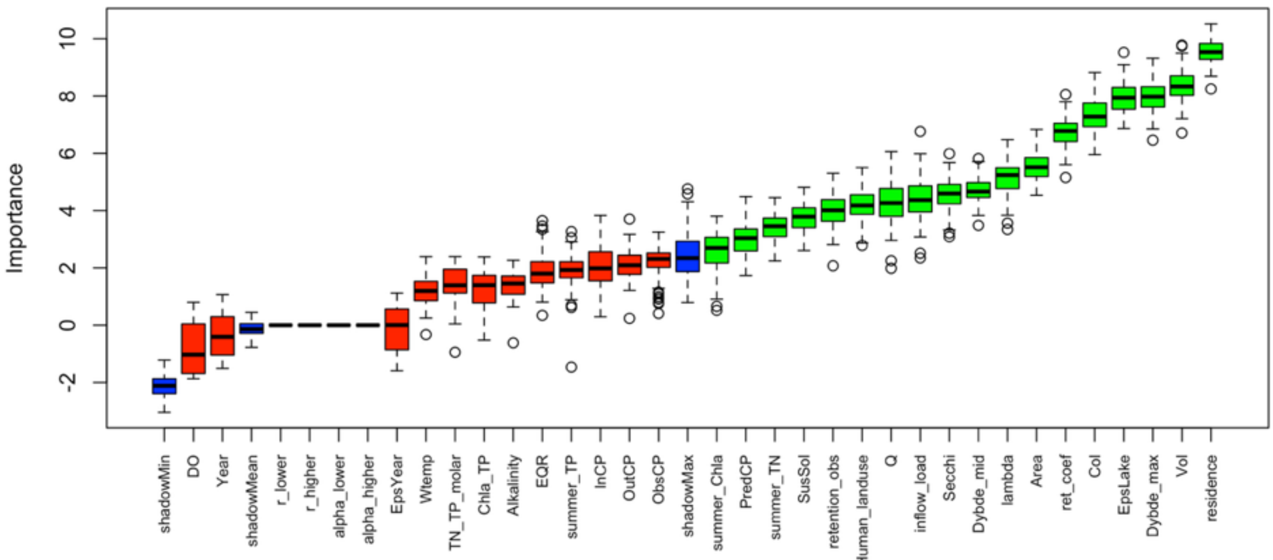


Figure 3. The importance of multiple water quality, lake characteristics, catchment and land use covariates on normalized gross TP retention identified using Boruta analysis. In the boxplots, horizontal solid line indicates median of the distribution, the box represents the lower to upper quartile values of the data, the whiskers extend to the last data point beyond 1.5 * the interquartile range, circles represent outliers beyond this range.

According to the Boruta analysis, an exponential model of normalized gross TP retention to residence time, $\overline{ret_{gross}} = a\, t_w{}^b$, was fitted to develop a simpler relationship for subsequent use as part of the SGAV sub-catchment model (Fig. 4). Compared to the previous exponential models of gross TP retention to discharge and area, the exponential models of normalized gross TP retention to residence time has lower R^2 values. Still, the updated exponential models have an improved R^2 compared to Trolle et al. (2015) (Table 1). General relationships are the same, i.e., TP retention in deeper lakes scales more strongly with residence time than TP retention in shallow lakes.

Table 1. Retention models fitted by Trolle et al. (2015) and in this report, in which λ is the TP retention, and t_w as hydraulic residence time (in years).

Retention model	Trolle et al. (2015)	R^2	This report	R^2
All lakes	$\lambda = 0.371 t_w{}^{0.1477}$	0.19	$\lambda = 0.2280916\, t_w{}^{0.07298647}$	0.46
Shallow lakes	$\lambda = 0.299 t_w{}^{0.0767}$	0.23	$\lambda = 0.1981346\, t_w{}^{0.01245211}$	0.55
Deep lakes	$\lambda = 0.406 t_w{}^{0.1206}$	0.23	$\lambda = 0.2106041\, t_w{}^{0.2079472}$	0.7

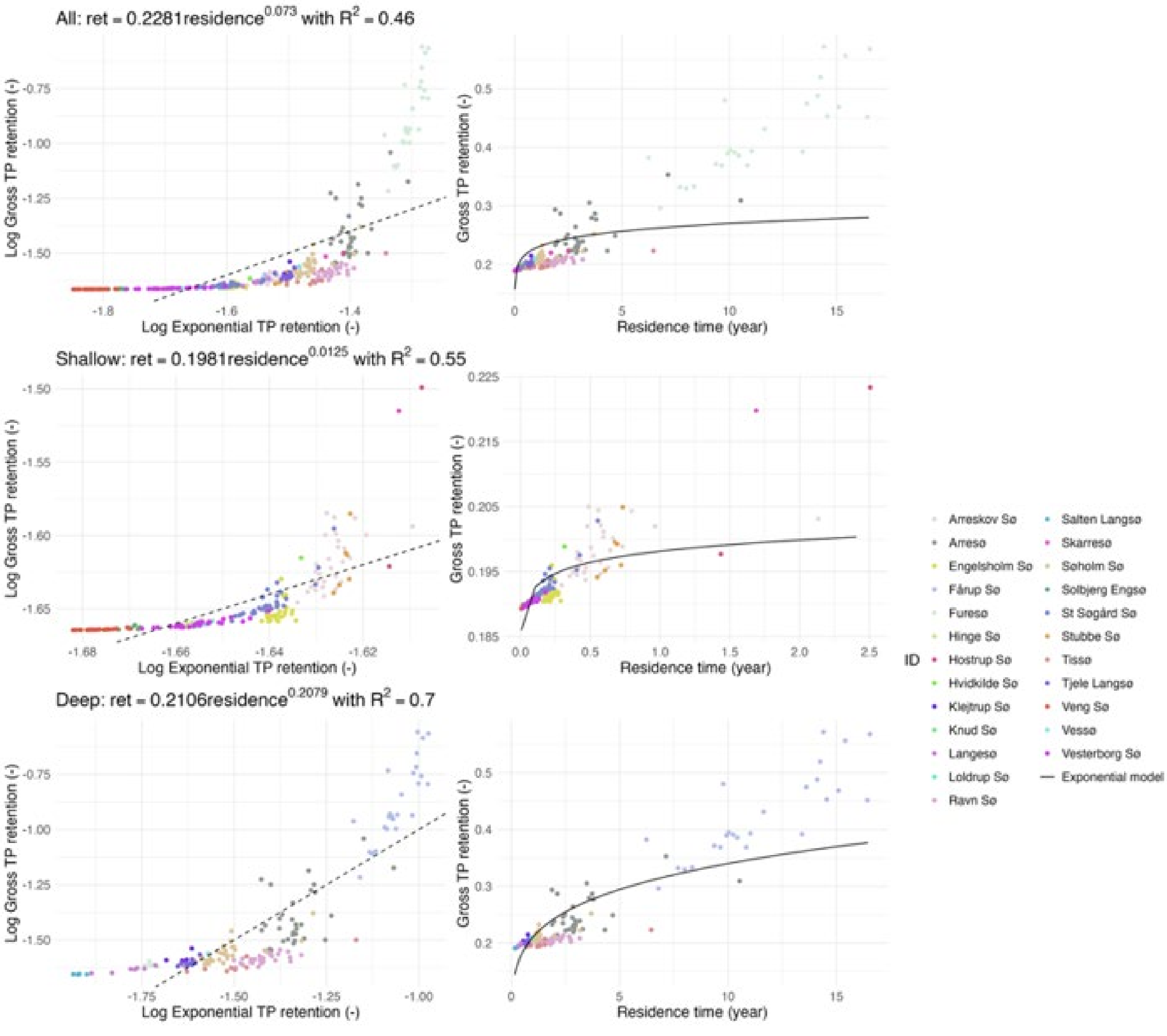
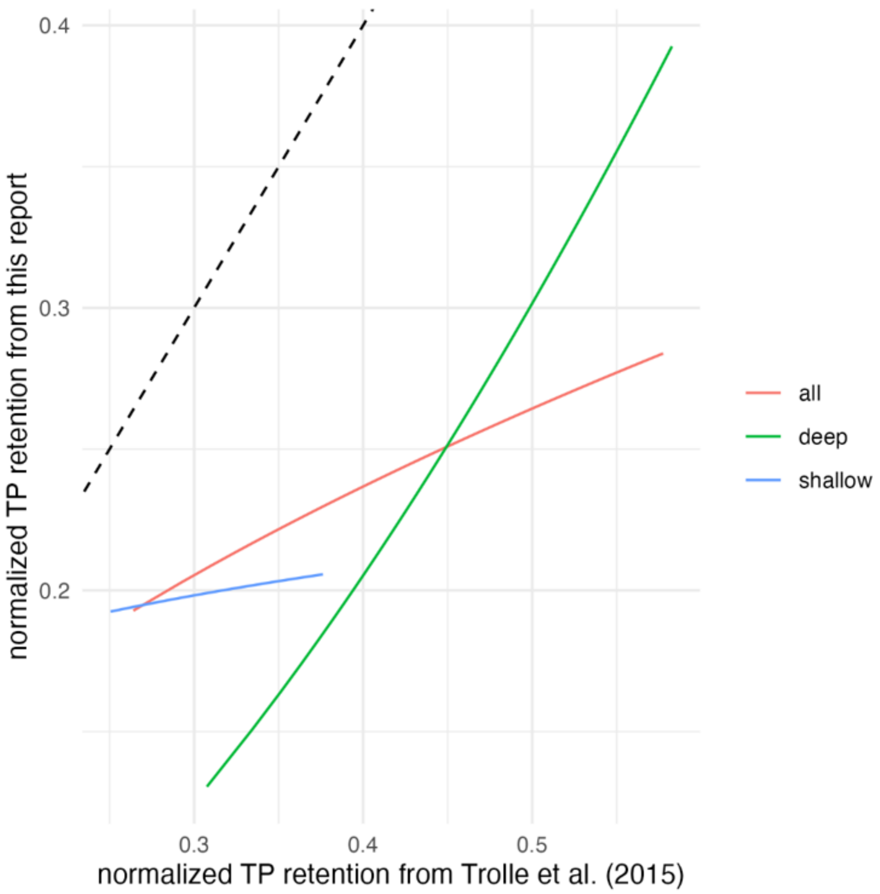


Figure 4. Left column: log-transformed normalized gross TP retention (y-axis) compared to predicted normalized gross TP retention by an exponential model based on residence time (x-axis)), for all lakes, shallow lakes and deep lakes. The equation above the plots is the derived exponential model. Plots in the left column include all lake-years, plots in the right column show normalized gross TP retention over residence time with the black curve representing the exponential model

A comparison with Trolle et al. (2015) indicates that the statistical model developed in this report generally predicts lower normalized TP retention than the earlier model (Fig. 5). Across all lakes included in the analysis (n = 24), the ratio between gross TP retention estimated by the present model and that estimated by Trolle et al. (2015), evaluated along a theoretical residence time gradient, is slightly above 0.5. In other words, the updated model predicts approximately half the retention of the previous model. The ratio is about 0.65 for deep lakes and 0.57 for shallow lakes. When applied within the SGAV subcatchment model, these results suggest a scaling factor (k) of approximately 0.57 for most lakes, although this comparison does not account for differences among years or other lake-specific conditions. The predicted lower retention by the updated model is probably due to (a) the fitting of the model to longer time series data, and (b) the incorporation of a TP loss term near the inlet. But, as the exponential model agrees well with the modeled gross TP retention (R^2 = 0.46 for all lakes), and the gross TP retention model achieved an RMSE of 0.038 mg TP L^{-1}, an NSE of 0.758, and an R^2 of 0.771 to observed TP, it can be assumed that observed TP retention in Danish lakes is lower than previously assumed.

Figure 5. Comparison of normalized TP retention from Trolle et al. (2015) to this report for a sequence of residence time ranging from 0.1 to 20 years. Dashed line represents a 1:1 intercept.



5 References

Ladwig, R., Søndergaard, M., Sørensen, P.B., Bucak, T. & Johansson, L.S.. 2026. Updated phosphorus retention model for Danish lakes. A note on statistical modeling of total phosphorus retention under steady-state assumptions. Aarhus University.

Trolle, D., Søndergaard, M. and R. Bjerring (2015). Sammenhænge mellem næringsstof-tilførsel og søkoncentrationer i danske søer. Aarhus Universitet, DCE – Nationalt Center for Miljø og Energi, Videnskabelig rapport nr. 138.